Introduction to Vectors

Week 1, Lesson 2

- **Vectors & Scalar Quantities**
- **Addition of Vectors**
  - **Graphical Addition**
  - **Parallelogram Method**
  - **Rectangular Components**
- **Subtraction of Vectors**

References/Reading Preparation:
Schaum's Outline Ch. 1
Principles of Physics by Beuche – Ch.1

❖ Scalar Quantities

Whenever we measure a quantity, we express our result in terms of a number.

For example, there are 50 students in this class.

This quantity has both a
- Numerical value (50), and
- A unit of measure (persons)

We can measure our height (165 cm, or 65 in or 5.4 ft)

In all cases, the quantity has a magnitude and a unit of measure – there is no direction associated with them.

Quantities that have no direction associated with them are called *Scalar Quantities*.

❖ Vector Quantities

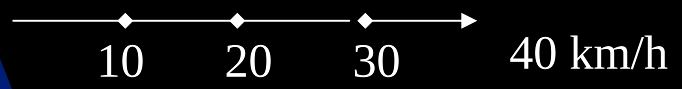
In contrast, a *vector quantity* has both a **magnitude** and a **direction**.

For example, motion is a quantity that involves direction as well as magnitude – such as a car traveling 40 km/h eastward.

Other vector quantities are forces and the movement one must undergo to travel from one city to another.

A vector quantity can be represented by an arrow drawn to scale.

For example, 40 km/h eastward can be represented by and arrow drawn to scale pointing east:



The length of the arrow is proportional to the magnitude of the vector quantity.

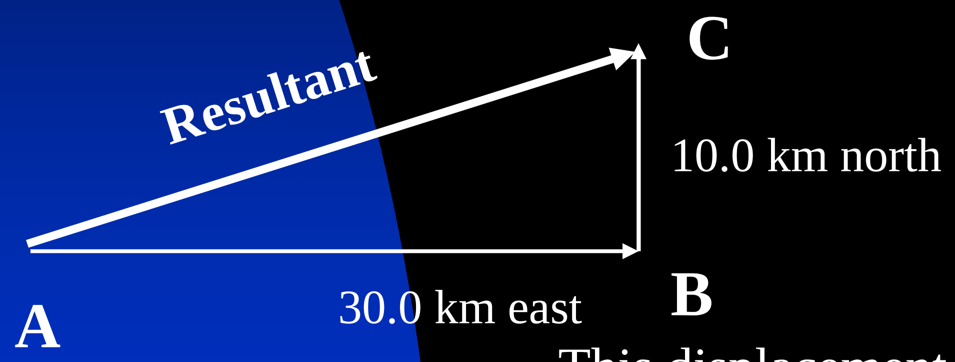
The direction of the arrow represents the direction of the vector quantity.

❖ Vector Addition

The sum of two ***scalars*** is simply the sum of their two magnitudes – they add numerically.
i.e : Adding 1600cm3 of water to 200cm3 of water gives 1800cm3.

Vector quantities do not add this way.

Example: 30.0 km east plus 10.0 km north



We are interested in the total **displacement** resulting from these two displacements- the displacement from A to C.

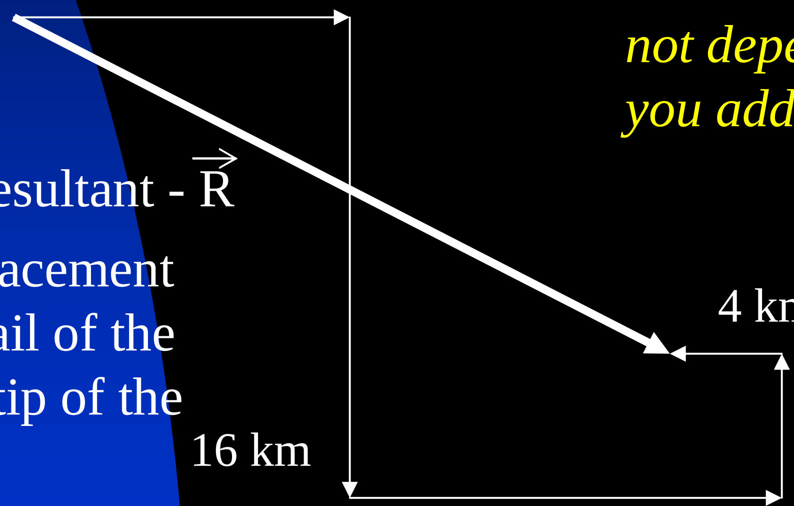
This displacement is called the Resultant displacement.

Note: its magnitude is $\sqrt{30^2 + 10^2} = 31.6km \approx 32km$

❖ Graphical Addition

Example – Find the resultant of 10 km east, 16 km south, 14 km east, 6 km north and 4 km west.

Solution: We can solve this graphically by drawing each vector to scale and successively adding them to find the resultant vector.

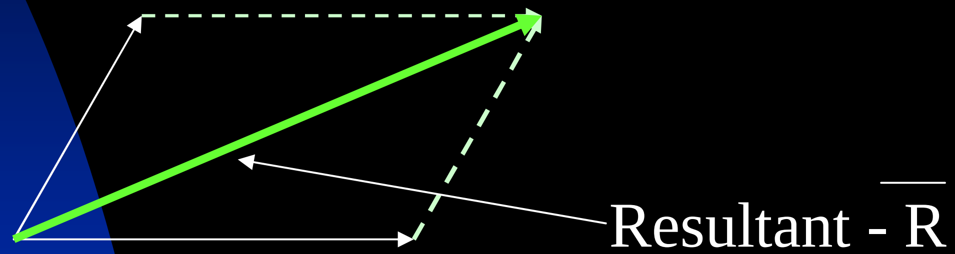


The result of adding vectors does not depend on the order in which you add them.

The resultant displacement extends from the tail of the First vector to the tip of the last vector.

❖ Parallelogram Method for Adding Two Vectors

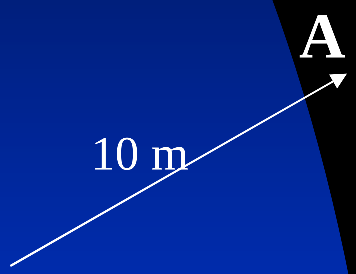
For two vectors acting at an any angle, the resultant can be represented by the diagonal of a parallelogram.



❖ Vector Subtraction

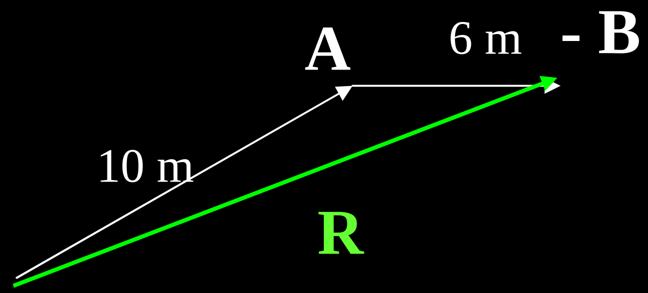
To subtract a vector, reverse the direction of the one being subtracted, then add.

For example, to subtract vector **B** from vector **A**, reverse the direction of **B** and then add it to **A** .
In math terms: **A – B** = **A + (-B)**



Vector A

Vector B



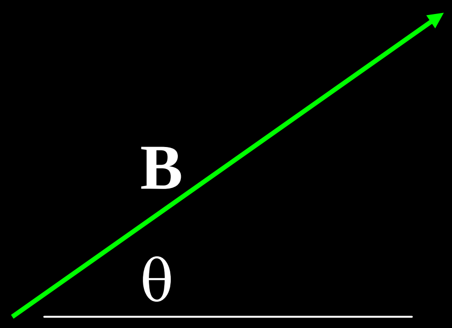
❖ Rectangular Components of Vectors

Although the graphical method for adding vectors is simple and straightforward, it is cumbersome and only as accurate as our scale drawings.

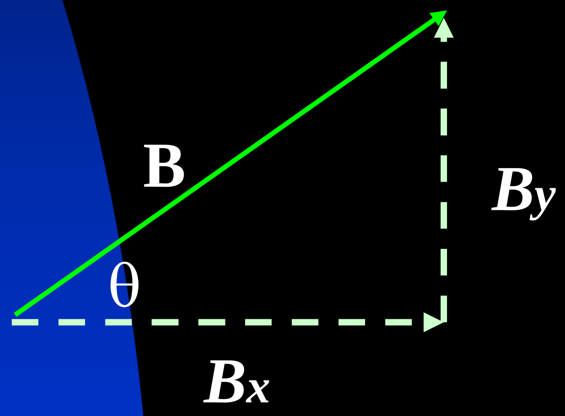
Therefore, another method is needed that that does not have these drawbacks.

First, we must learn how to find rectangular components.

Suppose we have the following vector.



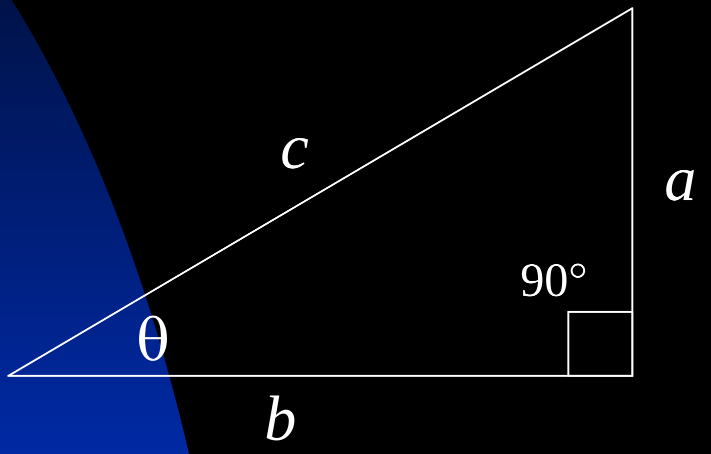
This vector can be replaced by two component vectors.



The displacement vector **B** can be replaced by the two vectors **A** and **C** which are at right angles to each other.

We call these two vectors the ***Rectangular Components*** of the original vector.

Before we go further, let's review the simple trigonometric functions of a right triangle.



$$\sin\theta = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{a}{c}$$

$$\cos\theta = \frac{\text{adjacent side}}{\text{hypotenuse}} = \frac{b}{c}$$

$$\tan\theta = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{a}{b}$$

It follows from these equations that the two sides can be found if we know the hypotenuse and one angle.

$$a = c\sin\theta \qquad b = c\cos\theta$$

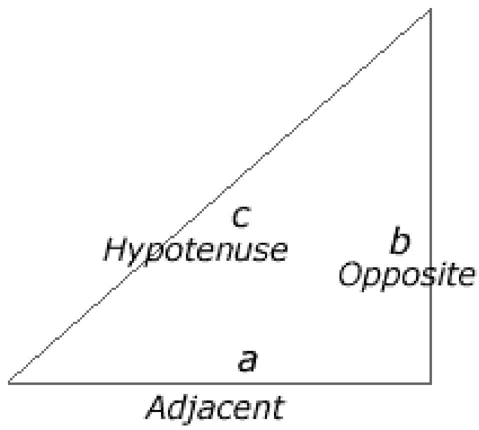
Trigonometric Formula

Useful trig formulas and some very helpful links for learning trigonometric concepts.

$$\text{Sin } A = \frac{\textit{Opposite}}{\textit{Hypotenuse}}$$

$$\text{Cos } A = \frac{\text{Adjacent}}{\textit{Hypotenuse}}$$

$$\text{Tan } A = \frac{\textit{Opposite}}{\textit{Adjacent}}$$



Triangle ABC is any triangle with side lengths *a*,*b*,*c*

Law of Cosines

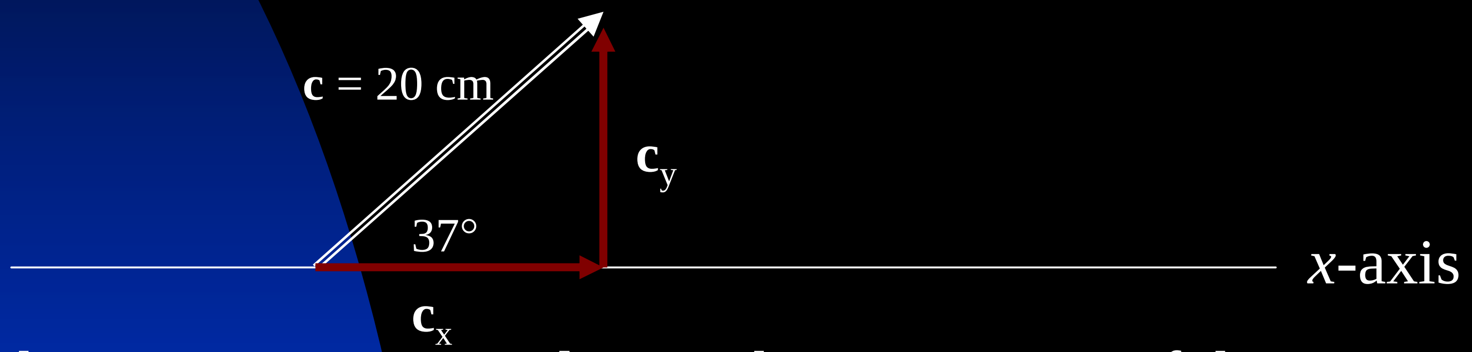
$$c^2 = a^2 + b^2 - 2ab \cos C$$

Law of Sines

$$\frac{\text{Sin } A}{a} = \frac{\text{Sin } B}{b} = \frac{\text{Sin } B}{c}$$

Let us apply this to finding the components of a vector.

The following figure shows a 20 cm displacement that makes an angle of 37° with the x-axis.



This vector **c** is equivalent to the vector sum of the two component vectors $\mathbf{c}_x$ and $\mathbf{c}_y$.

Where: $\mathbf{c}_x = \mathbf{c} \cos 37° = (20 \text{ cm})(0.80) = 16 \text{ cm}$

$\mathbf{c}_y = \mathbf{c} \sin 37° = (20 \text{ cm})(0.60) = 12 \text{ cm}$

The 20 cm displacement at an angle of 37° to the x-axis is equivalent to the sum of two rectangular component vectors:

$\mathbf{c}_x$ = 16 cm in the positive x-direction, and

$\mathbf{c}_y$ = 12 cm in the positive y-direction

It is possible in this way to to replace any vector by its rectangular components – in this way it is a simple matter to add, or subtract, vectors of all types.

❖ Component Method for Adding Vectors

Each vector is resolved into its *x*-, *y*- *and z*-components, with negative directed components taken as negative.

The x-component $\mathbf{R}_x$ of the resultant vector **R** is the algebraic sum of all the x-components. The *y*- and *z*-components of the resultant are found in a similar way.

With the components known, the magnitude of the resultant is given by:

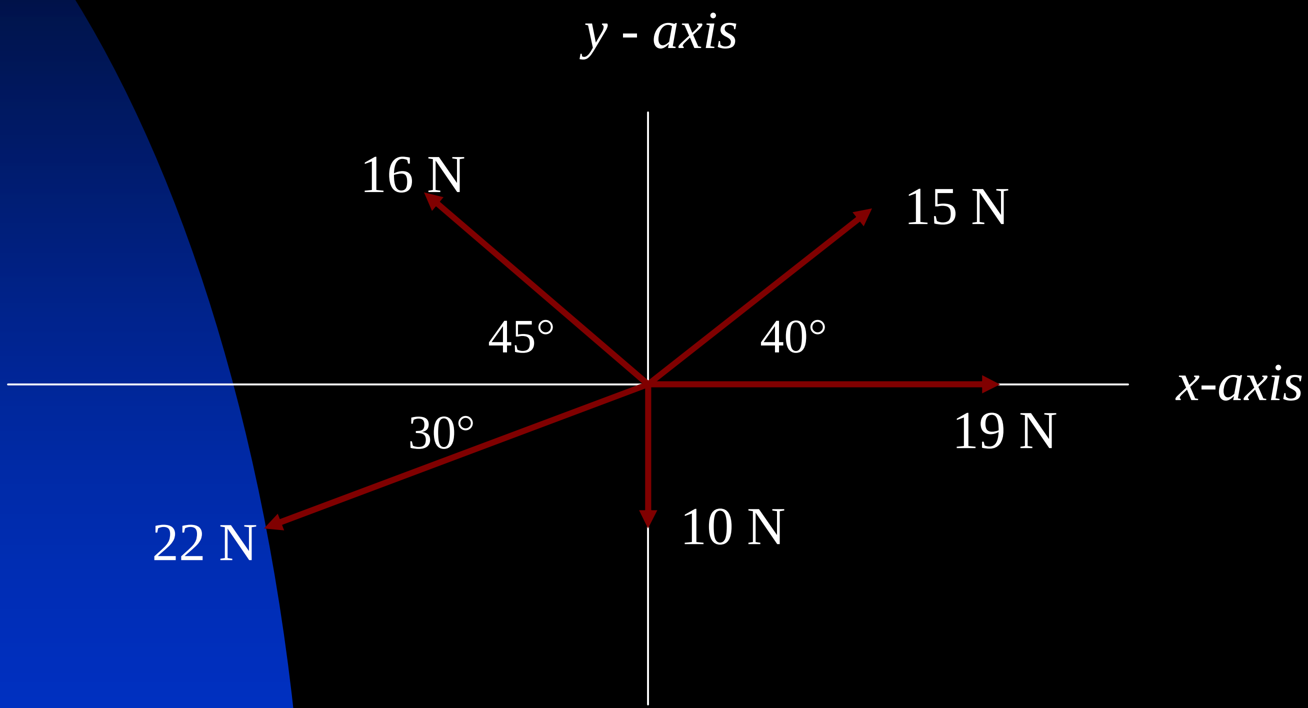
$$\boxed{R = \sqrt{(R_x^2 + R_y^2 + R_z^2)}}$$

In two dimensions, the angle of the resultant with the x-axis is:

$$\boxed{\text{Tan}\ \theta = R_y/R_x}$$

Example:

Five coplanar forces act on an object as shown. Find their component.



Solution:

Vector	***x*-component**	***y*-component**
19 N		
15 N		
16 N		
22 N		
10 N		